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A rapidly varied flow phenomenon in a two-layer flow

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This paper examines a region of rapidly varied flow in a two-layered density stratified system with one layer flowing, and the other stationary. The analogous phenomenon in open channel hydraulics is the hydraulic jump. In density stratified flows the phenomenon is referred to as a density jump because it is generally accompanied by a change in density of the flowing layer.

It is shown there is a fundamental difference between the hydraulic jump and the density jump in that flow conditions on either side of a density jump are not uniquely related. A density jump with given flow conditions upstream has a range of possible states which may be attained downstream. The rate of entrainment of ambient fluid into a density jump, and the conditions downstream of the jump, are determined by the downstream control and the upsteam conditions. The particular case of a density jump controlled by a broad crested weir downstream is examined in detail.

1. Introduction

This paper examines the characteristics of a hydraulic jump associated with some form of downstream control in a two-layer system. The fluids are of different densities and are miscible, so that mixing of the two layers may occur in the turbulent region of the jump. Since such mixing will result in a change in density of the flowing layer, the region of rapidly varied flow will be referred to as a density jump. The mixing region or entrainment zone of a density jump, has been studied experimentally by Ellison & Turner (1959).

The special case of a density jump occurring without mixing has been examined by Yih & Guha (1955). Their analysis showed that if both layers are flowing, up to four mutually conjugate downstream states are possible for a given upstream state. A unique solution is obtained when the downstream control is incorporated into the analysis.

Density jumps occur when a fluid is discharged from an outlet under a horizontal boundary into a fluid of greater density (figure 1, plate 1) or over a horizontal boundary in a fluid of lesser density. A situation similar to the former case exists when heated cooling water is discharged from a power station onto the surface of a cooling pond. Density jumps have also been observed in the atmosphere; Schweitzer (1953) observed the phenomena in Föhn winds, and Ball (1959) and Lied (1964) noted their occurrence in katabatic winds in Antarctica. Lied gives a particularly vivid account of his experience of walking through one of these jumps. In one instance a pressure change of nearly 20 mb was measured across the density jump, a distance of some 60 yards. However, pressure changes of 2–3 mb were more usual.

2. The mechanics of a density jump

Density jumps are similar in many ways to open channel hydraulic jumps. They are both transitions from a supercritical to a subcritical flow régime. However, there are several points of difference between the two phenomena. One of these is the ability of the density jump to entrain the varying amounts of ambient fluid necessary to satisfy a range of possible downstream controls. This characteristic of a density jump can be seen by comparing figures 1 and 2

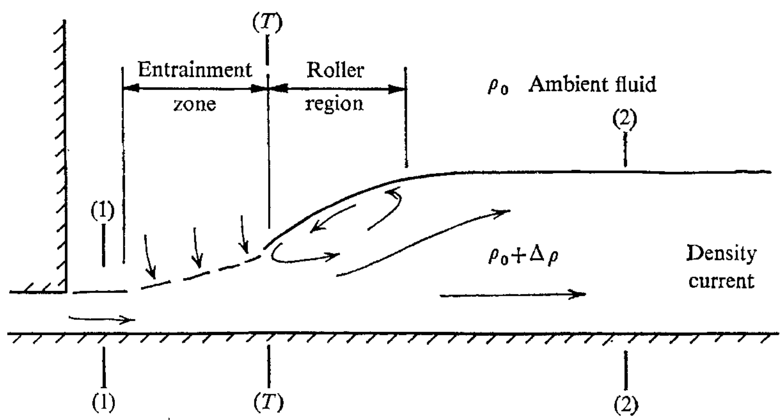

FIGURE 3. Schematic diagram of a density jump.

(plate 1), where photographs of density jumps with identical upstream states are shown but where each jump has a different downstream control. The difference in the jump shape is apparent.

It is necessary to examine the mechanism of a density jump in some detail before the interaction between a density jump and its control can be fully appreciated.

A density jump, in general, can be divided into two distinct zones; an entrainment zone and a roller region. Nearly all the entrainment which occurs at the jump takes place in the entrainment zone. These two zones are shown schematically in figure 3.

The region of entrainment and mixing occurs at the upstream end of a density jump and the entrainment mechanism in this zone is similar to that of a neutral wall jet. The length of the entrainment zone is determined by the downstream control. The presence of a control such as a channel contraction or a weir, as shown in figure 4, will cause a roller region to form at the downstream end of the jump. The roller region is characterized by a flow near the interface in the reverse direction to the main flow. This roller is similar in appearance to the roller associated with an open channel hydraulic jump and responds in a similar manner to a hydraulic jump if the height of a control weir downstream is adjusted. If the weir height (h) in figure 4 were increased, the roller region would extend further upstream and reduce the length of the entrainment zone. In experiments, the entrainment process was made visible by injecting dye patches into the ambient fluid in the vicinity of a density jump. It was observed that nearly all entrainment occurred in the entraining zone and that there is virtually no entrainment in the roller region of the density jump. It is apparent, therefore, that increasing the weir height will cause a decrease in the entrained flow at the density jump and hence the flow in the layer downstream of the jump will be reduced. The

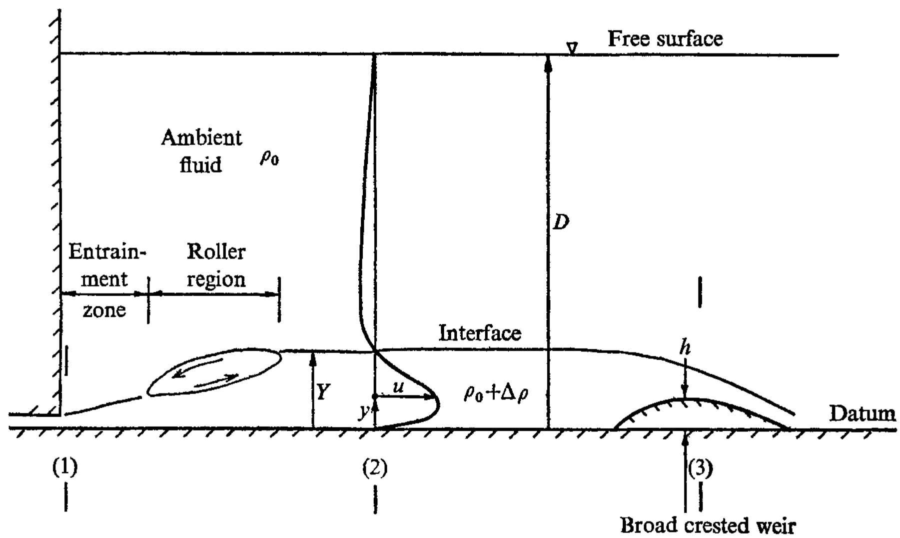

FIGURE 4. Density jump controlled by a broad-crested weir.

velocities in the region of reverse flow were far less than those in the entraining zone of the density jump. As a result, the interfacial shear is low in the roller region and the density difference between the two fluids is sufficient to inhibit turbulence and entrainment.

If the weir height is further increased, a point can be reached where the roller region extends the full length of the jump and entrainment effectively ceases. Such a jump will be referred to as a 'non-entraining jump'. Further increase in the weir height at this stage will cause the jump to flood so that the upstream end of the jump becomes submerged in dense fluid. There is no entrainment into a flooded jump. A photograph of a flooded jump is shown in figure 5 (plate 1).

If a controlling weir downstream of a density jump is lowered, the roller region can be observed to migrate downstream. This causes the entrainment zone to lengthen, and entrainment to increase, so that the flow in the layer downstream of the jump is also increased. A limit is reached when the entrainment zone occupies the entire length of the density jump, which is then of the maximum entraining type with no roller region. Such a density jump is shown in figure 1 (plate 1).

3. Analysis

The roller region, which is controlled by the downstream weir, affects the properties of the fluid flowing downstream of the jump and over the weir. The problem then is to predict the properties of the fluid between the jump and the weir (section (2), figure 4) given the properties of the inflowing fluid (the velocity, depth and density excess at section (1), figure 4) and the height of the weir (section (3), figure 4).

The method of solution makes use of the fact that the flow at all depths within the layer at sections (1), (2) and (3) is horizontal and hence the pressures are hydrostatic. It assumes that friction on the bed surface is negligible, the density excess values are small, and that there is no entrainment between sections (2) and (3). Use is then made of the equation of continuity of density excess and the balance of hydrostatic pressure forces and fluid momentum (this being termed the flow force by Benjamin (1962)), between sections (1) and (2). The equation of continuity and the equation for the flux of energy is used between sections (2) and (3). Finally it is necessary to use the fact that at the crest of the weir the flow is critical (Henderson 1966, pp. 40–44).

Let the density of the fluid at any point be $\rho_0 + \Delta\rho$ where ρ_0 is the density of the ambient fluid and $\Delta\rho$ is the density excess. Let u be the horizontal velocity and y the distance measured above the horizontal bed. Let D, the depth of the ambient fluid, be very much greater than the depth of the flowing layer, Y.

Let the flux of mass per unit span, the flow force per unit span and the energy flux per unit span be respectively, q_m, q_F and q_E. Then

$$\begin{aligned}\frac{q_m g}{\rho_0} &= \frac{1}{\rho_0}\int_0^D (\rho_0 + \Delta\rho)\, gu\, dy \\ &= g\int_0^D u\, dy + \int_0^D \frac{\Delta\rho}{\rho_0} gu\, dy \\ &= gq_v + q_\Delta, \end{aligned} \tag{1}$$

where the subscripts v and Δ refer respectively to the volume flux per unit span and to the flux of density excess times gravity per unit span. Since the flow is steady q_m, q_v and q_Δ must be constant at all sections. At the sections being considered the flow at all depths is horizontal and hence the pressures are hydrostatic. It follows that the flow force is given by

$$\frac{q_F}{\rho_0} = \frac{1}{\rho_0}\int_0^D (\rho_0 + \Delta\rho)\, u^2\, dy + \tfrac{1}{2}gD^2 + \int_0^D \int_y^D \frac{\Delta\rho}{\rho_0} g\, dy\, dy. \tag{2}$$

For the case where the datum from which the energy is measured is h below the bed, the energy flux is given by

$$\begin{aligned}\frac{q_E}{\rho_0} = {}& \frac{1}{2\rho_0}\int_0^D (\rho_0 + \Delta\rho)\, u^3\, dy + \frac{1}{\rho_0}\int_0^D (\rho_0 + \Delta\rho)\, g(y+h)\, u\, dy \\ & + \frac{1}{\rho_0}\int_0^D u \int_y^D (\rho_0 + \Delta\rho)\, g\, dy\, dy. \end{aligned} \tag{3}$$

Further let the volume flow in the layer alone be

$$q_L = \int_0^Y u\,dy,$$

where $y = Y$ is the position of zero horizontal velocity. It is important to note that q_L will differ from q_v by the induced flow in the ambient fluid. Although the induced flow may be of the order of q_L, because of the large depth in the ambient fluid, all the velocities outside the flowing layer are very small.

It is now proposed to assume that at all points $\Delta\rho \ll \rho_0$ and then to use the normal Boussinesq approximation. This implies that the density may be assumed constant and equal to ρ_0 in all terms except the body force terms. If now the characteristic depth, velocity and density excess times gravity are defined by Y, q_L/Y and q_Δ/q_L then substituting these values into (3) and assuming that outside the flowing layer the integrals such as

$$\int_Y^D u^2\,dy \quad \text{and} \quad \int_Y^D\int_y^D (\Delta\rho/\rho_0)\,g\,dy\,dy$$

tend to very small values we get

$$\frac{q_F}{\rho_0} = \frac{q_L^2}{Y}\int_0^1 u_*^2\,dy_* + \tfrac{1}{2}gD^2 + \frac{q_\Delta}{q_L}Y^2\int_0^1\int_{y_*}^1 \Delta_*\,dy_*\,dy_*, \tag{4}$$

where $$u_* = u/(q_L/Y), \quad y_* = y/Y, \quad \Delta_* = \left(\frac{\Delta\rho}{\rho_0}g\right)\bigg/\left(\frac{q_\Delta}{q_L}\right).$$

Defining $$\int_0^1 u_*^2\,dy_* = S_m,$$

$$\int_0^1\int_{y_*}^1 \Delta_*\,dy_*\,dy_* = \tfrac{1}{2}S_H,$$

and a layer Froude number as

$$F = (q_L^3/q_\Delta Y^3)^{\frac{1}{2}},$$

equation (4) can be written as

$$\frac{q_F - \frac{1}{2}\rho_0 g D^2}{\rho_0 q_\Delta^{\frac{1}{3}}} = \frac{q_L(2F^2S_m + S_H)}{2F^{\frac{4}{3}}}. \tag{5}$$

It is worth noting in (5) that S_m and S_H are measures of the non-uniformity of the velocity and density distributions respectively. If velocity and density throughout the layer depth Y are constant then S_m and S_H equal one.

Assuming velocities in the ambient fluid are sufficiently small for the free surface to be horizontal, i.e. D is a constant, then the above equation has the same value at sections (1) and (2). Hence one finds

$$\left|\frac{2F^2S_m + S_H}{F^{\frac{4}{3}}}\right|_1 = q_{21}\left|\frac{2F^2S_m + S_H}{F^{\frac{4}{3}}}\right|_2, \tag{6}$$

where the bracket subscript refers to the particular section and q_{21} is the ratio of the layer flows at sections (2) and (1).

In a similar manner the energy flux at any section can be written

$$\frac{q_E - \rho_0 g(D+h)\,q_v}{\rho_0 q_\Delta} = \frac{1}{2}\frac{q_L^3}{q_\Delta Y^2} S_E + S_p\, Y + h. \tag{7}$$

where
$$S_E = \int_0^1 u_*^3\, dy_*$$
$$S_p = \int_0^1 u_* \int_0^1 \Delta_*\, dy_*\, dy_* + \int_0^1 \Delta_*\, y_*\, u_*\, dy_*.$$

Again S_E and S_p are measures of the non-uniformity of the velocity and density distributions respectively. Since there is no entrainment between sections (2) and (3) and if it is assumed that the energy flux is constant, then the terms on the left-hand side of the equation are the same at each section. Further $Y_2/Y_3 = F_3^{\frac{2}{3}}/F_2^{\frac{2}{3}}$. Hence from (7) one can show

$$\left|\frac{F^2 S_E + 2S_p}{F^{\frac{2}{3}}}\right|_2 = \left|\frac{F^2 S_E + 2S_p + 2h/Y}{F^{\frac{2}{3}}}\right|_3. \tag{8}$$

Following the normal open channel flow theory (Henderson 1966), and differentiating (7) with respect to x and assuming that S_E and S_p are constant in the region between (2) and (3), one finds at section (3) where $dh/dx = 0$,

$$|(F^2 S_E - S_p)\, dY/dx|_3 = 0.$$

Since dY/dx is finite at this section it follows

$$F_3^2 = S_p/S_E,$$

and (8) becomes
$$\frac{h}{Y_3} = \left|\frac{(S_p/S_H)^{\frac{1}{3}}\,(F^2 S_E + 2S_p) - 3F^{\frac{2}{3}} S_p}{2F^{\frac{2}{3}}}\right|_2, \tag{9}$$

but from the definition of the Froude number one obtains

$$\frac{Y_3}{Y_1} = \frac{q_1 F_1^{\frac{2}{3}}}{q_3 F_3^{\frac{2}{3}}}.$$

Substituting for F_3 and noting the between sections (2) and (3) there is no entrainment and that $q_2 = q_3$ one gets

$$Y_3 = Y_1 q_{21} \left|\frac{S_E}{S_p} F^2\right|_1^{\frac{1}{3}}. \tag{10}$$

Thus from (9)

$$\frac{h}{Y_1} = q_{21} \left|\frac{S_E}{S_p} F^2\right|_1^{\frac{1}{3}} \cdot \left|\frac{(S_p/S_E)^{\frac{1}{3}}\,(F^2 S_E + 2S_p) - 3F^{\frac{2}{3}} S_p}{2F^{\frac{2}{3}}}\right|_2. \tag{11}$$

Further substituting for q_{21} from (6) one obtains

$$\frac{h}{Y_1} f = \left|\frac{(F^2 S_E + 2S_p - 3F^{\frac{4}{3}} S_p^{\frac{2}{3}} S_E^{\frac{1}{3}})\, F^{\frac{2}{3}}}{2F^2 S_m + S_H}\right|_2, \tag{12}$$

where
$$f = \left|\frac{2F^{\frac{2}{3}}}{2F^2 S_m + S_H}\right|_1$$

and is a function of upstream conditions only.

For a density jump controlled by a broad crested weir of known height, conditions downstream of the jump can be predicted from (12), provided upstream conditions are known. Equation (12) is plotted in figure 6 for the idealized case where the density and velocity are constant within the layer downstream of the density jump. Several interesting points arise from examination of this figure. (*a*) When the weir is of zero height, and in the absence of friction or any other downstream control, the Froude number downstream of the jump will be unity. For this case it can be shown from (6) that q_{21} is a maximum when $F_2 = 1$ (in general q_{21} is a maximum when $F_2 = (S_H/S_m)^{\frac{1}{2}}$). This implies that the jump is

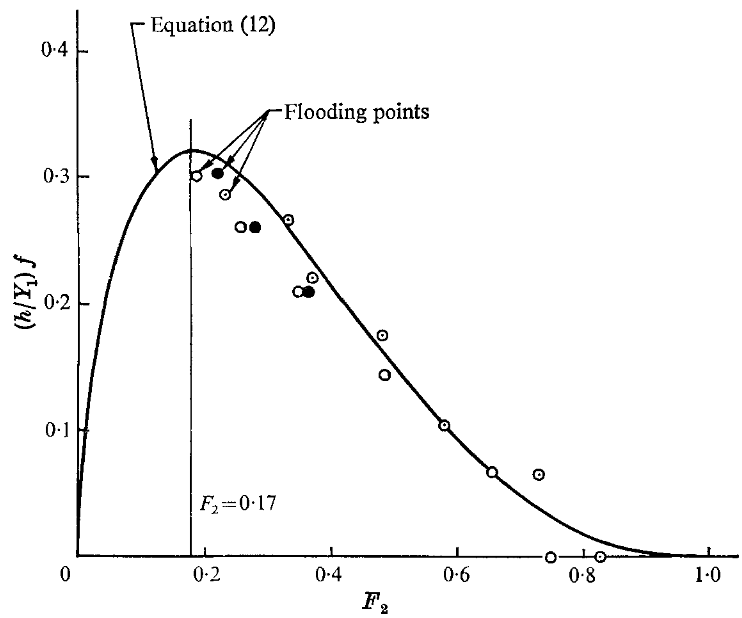

FIGURE 6. Plot of weir height against Froude number downstream of a density jump. Experimental results: ⊙, $F_1 = 10{\cdot}5$, h increasing; ⊕, $F_1 = 16{\cdot}5$, h increasing; ●, $F_1 = 16{\cdot}5$, h decreasing.

of the maximum entraining type. (*b*) There is a maximum value of $(h/Y_1)f$ for which a solution is possible. Since f and Y_1 are determined completely by upstream conditions, it follows that if the weir height is raised above this maximum value, the jump will flood. (*c*) It can be seen that for a density jump with set upstream conditions, and a given weir height downstream (i.e. f defined), two values of F_2 satisfy (12). It should be noted, however, that (12) only applies to unflooded density jumps, and once flooding occurs a different function relates $(h/Y_1)f$ and F_2.

This new relationship can be determined from (11) which equates flow energies at sections (2) and (3) in figure 4, and is independent of conditions at the density jump. There is no entrainment at a flooded density jump so that $q_{21} = 1$, and substituting into (11) for the idealized case where S_m, S_H, S_E and S_p are unity, one has

$$\frac{h}{Y_1} = \left|\frac{F^2 + 2 - 3F^{\frac{2}{3}}}{2F^{\frac{4}{3}}}\right|_2 F_1^{\frac{2}{3}}. \tag{13}$$

Both sides of (13) have been multiplied by f (defined in (12) and constant in value for given conditions upstream of a density jump) to enable (13) to be plotted on the same figure as (12) (figure 7).

The interrelation between (12) and (13) is now examined by means of an example. Consider first, a density jump, with an upstream Froude number of 3·0, controlled by a broad crested weir downstream. The curve for the flooded régime (13), of such a jump is plotted as the curve XY in figure 7. The point X represents the limiting condition where the roller region occupies the entire length of the

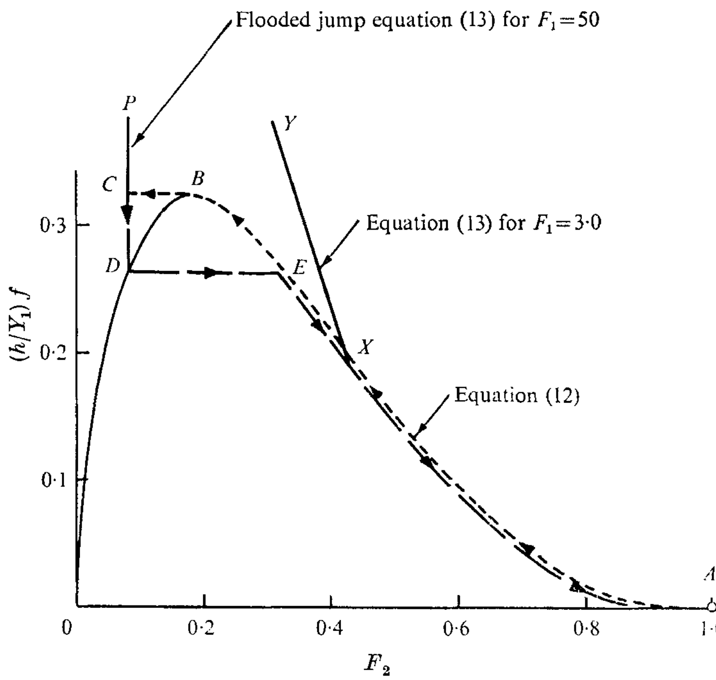

FIGURE 7. Effect of a weir raising and lowering cycle on a density jump.

jump, there is no entrainment and further increase in the depth of flow downstream of the jump will result in its flooding. This particular jump is non-entraining at the point X, and its downstream Froude number may be calculated by substituting 3·0 for F_1 in (6), to give $F_2 = 0·41$. Consider what happens as the weir is raised from zero height. Initially, the density jump will be of the maximum entraining type and the downstream state is given by the point A in figure 7. As the weir is raised h increases in value and it can be seen that F_2 decreases from its initial value of 1·0 at A. The curve $AXEBDO$ is followed until the point X is reached. Further increase in h at this point causes flooding of the jump and the downstream state is defined by the curve XY (i.e. equation (13)).

It is apparent that only a limited region of the curve $AEBDO$ will describe possible downstream states of a density jump. This region will depend on the value of the Froude number upstream of the jump. It will now be shown that no stable downstream states exist to the left of B on the curve $AEBDO$. This region is potentially accessible to the density jumps with an upstream Froude number of 13·2 or greater. Density jumps having upstream Froude numbers of less than 13·2 will flood before the downstream Froude number reaches 0·17, or before the point B is attained. It will be shown that for density jumps controlled by a broad crested weir Froude numbers of less than 0·17 cannot be attained, irrespective of the upstream Froude number. Once such a jump has reached a downstream Froude number of 0·17, further increase in weir height results in flooding of the density jump, and the resulting phenomenon is no longer considered to be a density jump. The physical form of a flooded jump (figure 5, plate 1) is quite different to that of a density jump (figures 1 and 2, plate 1).

The reasons for this limiting characteristic of density jumps controlled by broad crested weirs can be best understood by tracing a cycle of weir raising and lowering on a plot of $(h/Y_1)f$ versus F_2. This cycle is shown in figure 7 for a density jump having an upstream Froude number of 50. Commencing at a point A with a weir of zero height and a maximum entraining density jump, as the weir height is increased, the downstream Froude number (F_2) decreases in value. This process can be continued until the point B is reached. At this stage the density jump is still entraining ambient fluid.

Since no solutions for density jumps exist for values of $(h/Y_1)f$ greater than that at B, the smallest increase in weir height at this point results in flooding of the jump. A small increase in h at B results in the jump flooding to a state given by C on the curve PCD. This curve is equation (13) plotted for the case where $F_1 = 50$, and hence from the definition, $f = 0{\cdot}0054$.

This flooded state is quite stable, and it can be seen the inlet is submerged, as in figure 5. Lowering of the weir will not cause the density jump to return to its former state, given by B in figure 7. For this to happen all of the fluid overlying the inlet would have to be swept away. This will not occur as the flow is stable on PCD, if small quantities of overlying fluid are removed. The jump therefore remains flooded until the point D is reached.

It is now shown that a non-entraining jump to the left of B (i.e. at D) in figure 7 is unstable, and a negative disturbance of the weir height downstream will result in another dramatic change in the form of the jump. However, before proceeding further, it is necessary to examine a further characteristic of the density jump.

It was stated earlier that the non-entraining roller region of a density jump is analogous to the open channel hydraulic jump. If a control weir downstream of a hydraulic jump is lowered, the jump moves downstream until some new equilibrium is established. The roller region of a density jump behaves in an identical manner. When a roller region migrates downstream an increased length of entraining zone is exposed, so that the total entrainment is increased. The flow downstream of a density jump is therefore increased until equilibrium is restored.

It is now shown that this downstream migration of the roller region results in an increase in the Froude number downstream of a density jump. Let F_T denote the transitional Froude number at the junction of the roller region and the entrainment zone (section T–T in figure 3). A range of possible F_T values are available in any density jump between the limits $F_T = F_1$ at the upstream end of the jump, and $F_T = 1{\cdot}0$ in the idealized case of maximum entrainment. If the roller region moves downstream, it follows F_T must decrease in value. It has been stated the roller region is a non-entraining stationary surge, so that it may be analyzed in a similar manner to the open channel hydraulic jump. If it is assumed that velocity profiles either side of the roller region are reasonably uniform, and that the velocities are horizontal, then the equation of Bakhmeteff (1932, pp. 240–1) for hydraulic jumps will hold

$$F_2 = \frac{F_T}{\{\frac{1}{2}[(8F_T^2+1)^{\frac{1}{2}}-1]\}^{\frac{3}{2}}}. \tag{14}$$

Return now to figure 7 and examine the non-entraining jump at D. This is the transition point between a density jump and flooded jump. If the weir height is

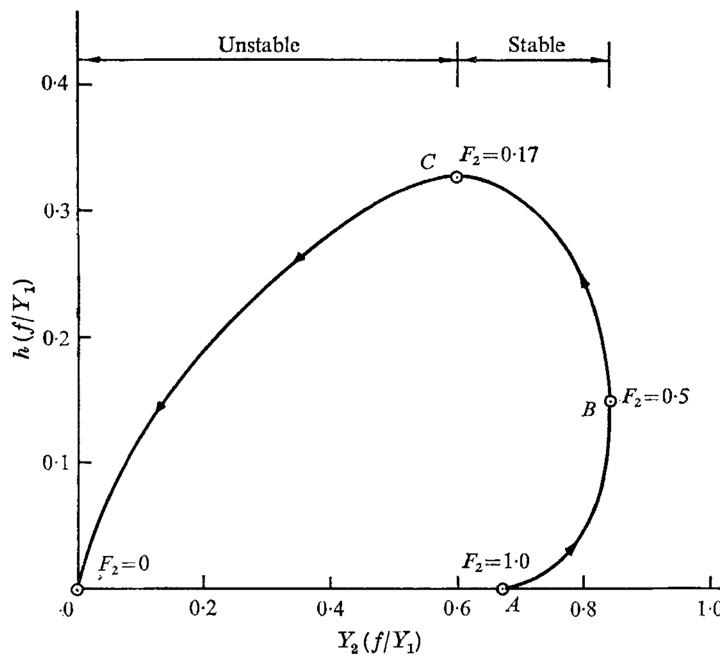

FIGURE 8. Weir height versus depth downstream of a density jump.

lowered slightly, the roller region moves slightly downstream, entrainment commences, F_T decreases and it can be shown from (14) that F_2 increases in value. It is apparent, however, that if F_2 is to increase in value, a new equilibrium cannot be attained until the point E is reached. Thus the non-entraining state of the jump at D is unstable, and a disturbance to the flow will result in a sudden change in the form of the jump. The flow will suddenly change to a density jump (i.e. jump in which the density changes) and there will be a rapid increase in the Froude number downstream of the jump.

An intriguing relationship exists between the depth of flow downstream of the density jump (section (2) in figure 4) and the height of the broad-crested weir.

Using the definition of the Froude number and the expression for q_{21} from (6) one obtains

$$\frac{Y_2}{Y_1} = \left|\frac{2F^2S_m+S_H}{F^{\frac{2}{3}}}\right|_1 \bigg/ \left|\frac{2F^2S_m+S_H}{F^{\frac{2}{3}}}\right|_2.$$

This relationship, together with equation (12), enables us to eliminate F_2 and hence obtain a relationship between Y_1 and Y_2 in terms of the upstream conditions. This relationship is plotted on figure 8.

Starting with a weir of zero height and a maximum entraining density jump at A, the depth downstream of the jump Y_2 increases as the weir height (h) is increased. Once, however, the point B is reached Y_2 has attained its maximum value. Beyond this point raising of the weir forces the roller upstream and causes such a decrease in discharge, and an increase in the density at section (2), that the depth upstream of the weir actually falls. This depth will continue to decrease until the point C is reached where it can be shown $F_2 = 0{\cdot}17$. This point corresponds to B in figure 7 and if h is increased beyond this value the density jump will flood.

The depth weir height relationship downstream of a density jump controlled by a broad-crested weir is quite novel; an increase in the weir height can result in a decrease in the depth of flow upstream of that weir.

4. Experimental results

Experiments were performed in the tank shown schematically in figure 9. Thermal density currents were used in experiments, the warm layer flowing over the cooler ambient layer as shown in the figure below. The dimensions of the

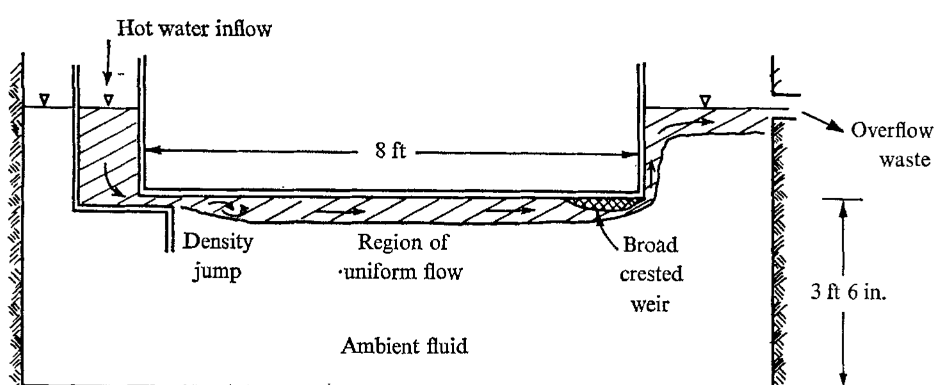

FIGURE 9. Schematic diagram of the test tank.

working section of the tank were 8 ft long by 6·1 inches wide by 3 ft 6 in. deep. The sides were of $\frac{3}{4}$ in. thick perspex so that heat transfer across the boundaries was negligible. Steady-state flows could be maintained indefinitely. Temperatures were measured using calibrated thermocouples and velocities were measured using a hydrogen bubble technique.

Mean values of the characteristic density difference and the integral flow force and energy terms, S_m, S_H, S_p and S_E, could be determined to an accuracy of 5 % or better, from these measurements.

Data from experiments with density jumps, controlled by broad-crested weirs, are plotted in figure 6. The theoretical relationship between the plotted parameters, the dimensionless weir height and the downstream Froude number, is also shown in this figure.

It can be seen that agreement between the theory and experiment is satisfactory. It should be noted, however, that the theoretical curve was plotted for the ideal case where it was assumed that the velocity and density distributions are uniform in the flowing layer. The closeness of the curve and the data indicated

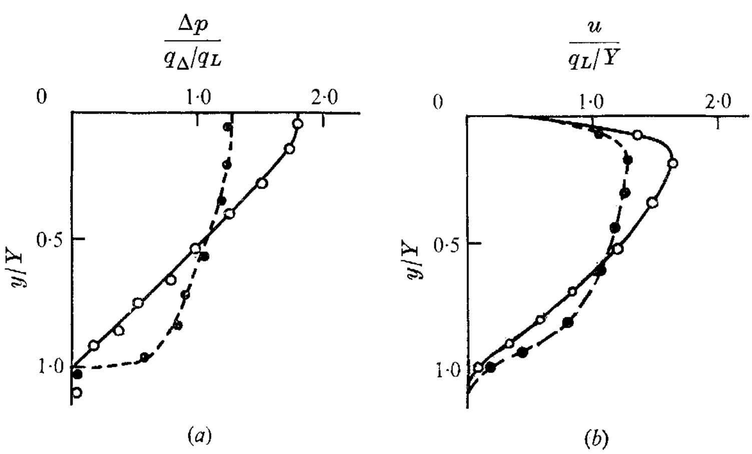

FIGURE 10. Typical velocity and density distributions downstream of a density jump ($F_1 = 7{\cdot}1$). (a) Mean density distribution. (b) Mean velocity distribution. – – –, partially entraining; ——, maximum entraining.

that the various integral factors compensate. (This conclusion was verified by detailed measurement of velocities and densities downstream of the density jump.)

The flooding points of density jumps having upstream Froude numbers of 10·5 and 16·5 are shown in figure 6. Unfortunately, with the experimental apparatus available, upstream Froude numbers greater than 16·5 could not be attained. It was not possible therefore to experimentally verify with any certainty the stability theory. The density jump having an upstream Froude number of 16·5 flooded before the non-entraining value of downstream Froude number was obtained. However, experimental accuracy was not sufficiently precise to observe the small hysteris effect which could be expected at this value of F_1.

4.1. *The downstream layer*

Dimensional analysis indicates that any density jump is fully defined by its upstream Froude and Reynolds numbers and its downstream Froude number. It follows therefore that the velocity and density distributions downstream of a jump can also be defined in terms of these three parameters. It was found that the Reynolds number effects were minor except close to the solid boundaries.

4.2. *Velocity distributions*

Typical mean velocity distributions measured downstream of a density jump, for cases of maximum and little entrainment, are shown in figure 10(*b*). The Froude number upstream of this density jump was 7·1 and the respective downstream Froude numbers were 0·73 and 0·38. The profile of the jump, for each case, is shown in figure 11, and the difference in form is apparent.

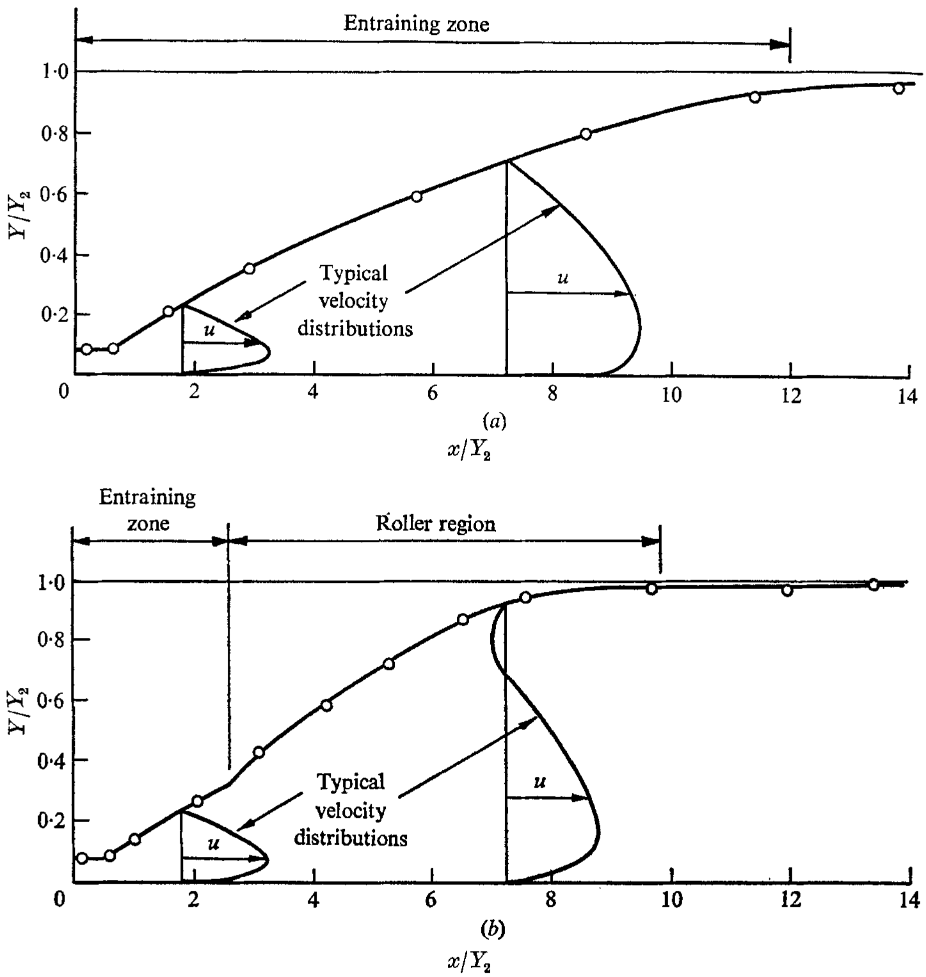

FIGURE 11. Profiles of density jumps. (*a*) Maximum entraining; (*b*) partially entraining.

Experiments showed that the velocity distribution, downstream of a density jump which had a constant upstream state, became more non-uniform as the downstream Froude number increased in value, that is as entrainment increased and the height of the control weir decreased. It was also found that the downstream velocity distribution was largely independent of the upstream Froude number, provided its value exceeded three.

The non-dimensional momentum term S_{m2} is a function of the velocity distribution in the downstream layer and is plotted against the downstream Froude number in figure 12. High values of S_{m2} (which implies a non-uniform velocity distribution) are associated with high values of downstream Froude number. The value of the upstream momentum term S_{m1} was found to vary from 1·08 to 1·14, depending upon the Reynolds number of the flow in the inlet.

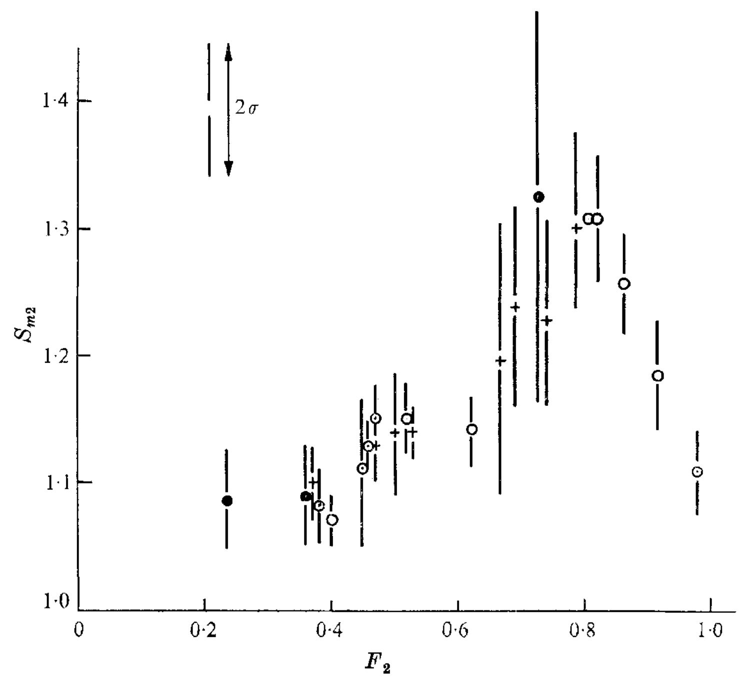

FIGURE 12. The integral momentum term versus downstream Froude number. ○, F_1 = 1–3; ⊕, F_1 = 3–5; +, F_1 = 5–10; ●, $F_1 > 10$.

4.3. *Density distributions*

The density distribution downstream of a density jump is determined by two factors: (i) the amount of entrainment; (ii) the length of the roller region (this determines the degree of mixing between the entrained fluid and the upstream layer).

For a density jump with given upstream conditions, the downstream density distribution will become more non-uniform as the downstream Froude number increases in value. This increase in Froude number, which would result from a decrease in height of the control weir, causes both an increase in entrainment and a reduction in the length of the roller region. An example of this may be seen in figure 10(*a*).

The density distribution downstream of a density jump with a given downstream control will become more non-uniform if the upstream Froude number is increased in value, since entrainment will also increase.

The non-dimensional pressure force term S_{H2} is a function of the density distribution in the downstream layer. S_{H2} is plotted as a function of upstream and downstream Froude numbers in figure 13. Non-uniformity in the density distribution causes a reduction in the value of S_{H2}.

High values of downstream Froude number are associated with high values of entrainment and non-uniform density distributions, so that S_{H2} for such a jump tends to be low valued. S_{H2} was found to have a minimum value of approximately 0·6.

The density distribution upstream of density jumps, was uniform in all experiments, hence S_{H2} was equal to one.

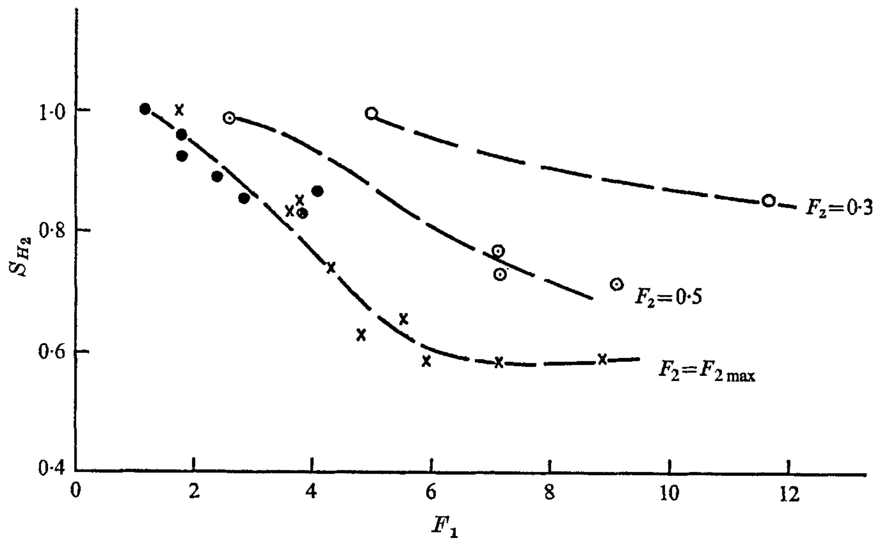

FIGURE 13. The integral pressure term as a function of upstream and downstream Froude numbers. ●, F_2 = 1·0–0·8; ×, F_2 = 0·8–0·6; ⊙, F_2 = 0·6–0·4; ⊕, F_2 = 0·4–0·2.

5. Conclusions

It has been shown that for a density jump controlled by a broad-crested weir two different values of downstream Froude number satisfy the equation of motion. Physical arguments have been used to show that only for the upper value of Froude number is the flow state stable. Although the lower value of Froude number can be achieved by first flooding the jump and then lowering the control weir until the jump becomes non-entraining, a slight decrease in the weir height at this point will result in a dramatic change in form of the density jump and the flow rate downstream. It was noted that for density jumps with downstream Froude numbers of less than 0·5, an increase in the height of the control weir caused a decrease in the layer depth upstream of the weir.

Other forms of control have been investigated (Wilkinson 1970); among these being contractions, undershot gates and friction on a sloping bottom. In all the above cases, the Froude number downstream of the density jump is a single-valued function of the control parameters and the Froude number upstream of the density jump.

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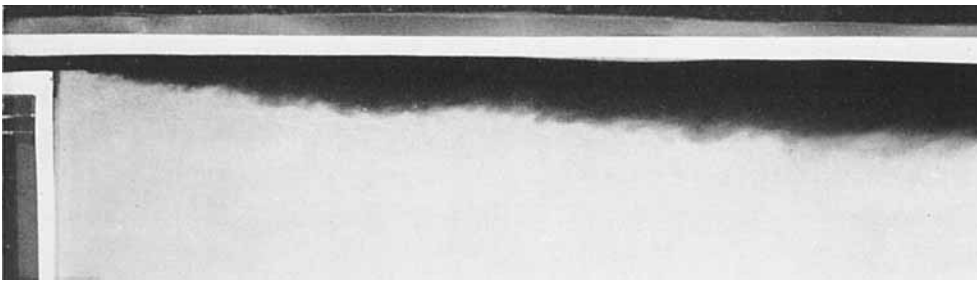

FIGURE 1. A density jump of the maximum entraining type with a free overfall downstream. Entrainment is occurring along the length of the jump.

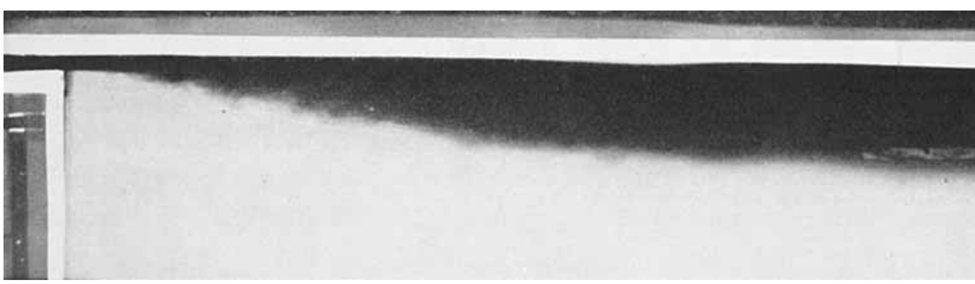

FIGURE 2. A density jump controlled by a broad crested weir downstream. Entrainment is occurring only at the far upstream end of the jump, the remainder consists of a roller zone.

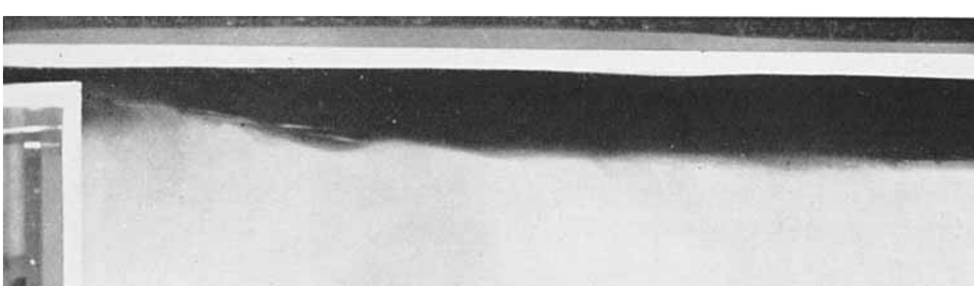

FIGURE 5. Photograph of a flooded density jump. The inlet flow for this jump is identical to that of the jumps shown in figures 1 and 2.